University of Toronto at Scarborough
CSC A67/MAT A67 - Discrete Mathematics, Fall 2015

Exercise #7: Proofs

Due: November 20, 2015 at 11:59 p.m.
This exercise is worth 3% of your final grade.

Warning: Your electronic submission on MarkUs affirms that this exercise is your own work and no one else's, and is in accordance with the University of Toronto Code of Behaviour on Academic Matters, the Code of Student Conduct, and the guidelines for avoiding plagiarism in CSC A67/MAT A67.

This exercise is due by 11:59 p.m. November 20. Late exercises will not be accepted.

1. The *greatest common divisor* of two positive integers a and b is the largest positive integer that divides both a and b (written $\gcd(a, b)$). For example, $\gcd(4, 6) = 2$ and $\gcd(5, 6) = 1$. [5]
   (a) Prove that $\gcd(a, b) = \gcd(a, b - a)$.
   (b) Let $r = b \bmod a$. Using part **(a)**, prove that $\gcd(a, b) = \gcd(a, r)$.
2. Prove that $\sqrt[3]{5}$ is irrational. [4]
3. Prove the following statement by contraposition: [4]
   Let x be an integer. If $x^2 + x + 1$ is even, then x is odd.
4. Prove the following statement by contradiction: [4]
   Let x and y be integers. If $3x + 5y = 153$, then at least one of x and y is odd.
5. An integer is called "sane" if $3 \mid (n^2 + 2n)$. (That is, if $(n^2 + 2n) \bmod 3 = 0$.) [4]
   (a) Prove or disprove that all odd integers are sane.
   (b) Prove or disprove that, if $3 \mid n$, then n is sane.

[Total: 21 marks]